

1. (a) (3 points) Using Euclidean algorithm, express $\gcd(134, 273)$ as an integer linear combination of 134 and 273. (That is, find integers s and t such that $\gcd(134, 273) = 134s + 273t$.)
(b) (3 points) Solve the linear congruence $134x \equiv 272 \pmod{273}$.
2. (6 points) Consider the summation $s_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)}$.
 - (a) Find s_1, s_2 , and s_3 .
 - (b) Conjecture a closed formula for s_n .
 - (c) Use mathematical induction to prove the formula you conjectured in part (b).
3. (6 points) We call a positive integer *perfect* if it equals the sum of its positive divisors other than itself (for example, 6 is perfect because $6 = 1 + 2 + 3$).
 - (a) Show that 28 is perfect.
 - (b) Show that 2^m is not a perfect number where m is a positive integer.
 - (c) Show that $2^{p-1}(2^p - 1)$ is a perfect number when $2^p - 1$ is prime.
4. (3 points) Show that 2^{2n} is not $O(2^n)$.
5. (a) (2 points) Find the prime factorization of $15!$.
(b) (2 points) Use part (a) to prove or disprove that $15!$ ends in exactly four 0s.
6. (7 points) Solve the system of congruences using the Chinese remainder theorem or the back substitution method:
$$\begin{cases} x \equiv 6 \pmod{11} \\ x \equiv 4 \pmod{13} \\ x \equiv 5 \pmod{19} \end{cases}$$

(Hint: The multiplicative inverse of 143 modulo 19 is 2.)
7. (4 points) Find the remainder when $2^{100} \cdot 3^{336} \cdot 5^{336} \cdot 101^{100}$ is divided by 233.
(Hint: 233 is a prime number, $2 \cdot 233 - 31 \cdot 15 = 1$, $233 - 15 = 202$.)
8. (4 points) Prove that if a is a positive integer, then 4 does not divide $a^2 + 2$.
(Hint: Think modulus 4.)
9. Take a photo of yourself holding the answer to the first page of your work and your photo ID (preferably your Langara ID). You must upload this photo as part of your assessment. Your photo should look like the diagram below.

